Oxford Mathematics Team Challenge
Guts Round Instructions

Saturday, 8^{th} March 2025

Instructions

1. Do not look at the first set of questions until the invigilator tells you to do so.

2. **Format.** This round contains nine sets of 3 questions each. You must submit your current set of questions in order to receive the next set. You cannot update your answers to any set once submitted.

3. **Time limit.** 80 minutes. You may not submit any answers after the allotted time has expired.

4. **No calculators, squared paper or measuring instruments.** Lined paper and blank paper for rough working is allowed. You may use a pen or pencil to preference. Other mediums for working (e.g. digital devices, whiteboards, augury) are strictly forbidden.

5. **Scoring rules.** For the first eight sets, your team is awarded full points for each correctly answer, and zero points otherwise. Answers must be fully simplified and in exact form; approximations will not be accepted. In the final set, which is the *estimathon*, your score for each question is $16 \times \frac{\min\{A,E\}}{\max\{A,E\}}$, rounded to the nearest integer, where A is the actual answer and E is your estimated answer.

6. Don't expect to solve every question in the time given! You may consider skipping to the estimation set at the end of the round, taking some time to attempt the questions to score partial credit.

7. You are also encouraged to think deeply, rather than to guess.

8. To accommodate the online version of the competition, please do not discuss or distribute the paper online until 08:00 BST on Monday 24^{th} March.

9. Good luck, and enjoy! ☺

Set	1	2	3	4	5	6	7	8	9
Points per question	3	4	5	5	6	6	7	8	16

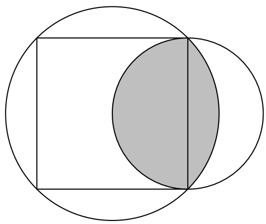
Set 1

Questions in this set are worth 3 points

1. Two countries each have an Olympic tennis squad of ten players. Each country chooses two of their players to compete in a doubles match. How many possible lineups are there for the match?

2. What is the third smallest positive integer whose digits sum to 25?

3. Jimbo is drawing a coffee bean. He first draws a square of side length 2, draws a circle whose diameter is a side of the square, and then draws another circle circumscribing the square. The intersecting region of the two circles is his coffee bean. What is the area of his coffee bean?



Set 2

Questions in this set are worth 4 points

4. Mr. Cheng needs to split his class of 7 students into 2 groups for a project, one group with 3 students and another with 4 students. Two of Mr. Cheng's students are Richard and Alice, who must be kept in separate groups because they quite strongly dislike each other. How many ways can Mr. Cheng form the two groups?

5. The rectangle $ABCD$ has side lengths $AB = 9$, $BC = 6$. Point P is chosen inside the rectangle such that the areas of $ABPD$, BCP and CDP are all equal. Find the length of AP.

6. The positive integer N satisfies $108^2 + 144^2 = 2025N^2$. What is the value of N?

Set 3

Questions in this set are worth 5 points

7. Let AB be the diameter of a semicircle, and let X lie on segment AB such that $AX = BX$. Choose points Y and Z on the arc of the semicircle such that $AY < AZ$. Let P be the intersection of AZ and XY, and suppose that $AP = 4$, $PX = 2$, $PY = 2$, and $PZ = 3$. Find AY.

8. Noah has four playing cards with the words NO, OH, AN, and HA written on their faces, each card having a different word. He randomly arranges the four cards face-down into a row and then flips them over revealing the words on them. What is the probability that the first card's word contains the letter N, the second word contains O, the third word contains A, and that the fourth word contains H? Express your answer as a common fraction.

9. Let S be the set containing the terms in the arithmetic sequence $1, 8, 15, \ldots$ that are less than 2000. Find the smallest n such that, for any n numbers picked from S, there always will be a pair of numbers picked that add up to 2025.

Set 4

Questions in this set are worth 5 points

10. Let P and R be two points on a circle of radius 5. Point Q is inside the circle. Suppose that $PQ = 1$, $QR = 7$, and $\angle PQR = 90^\circ$. The segments PQ and QR together split the circle into two regions. What is the area of the smaller region?

11. Each morning, Arav has to decide how to get from his home to his maths class, which is 5 miles due south of his home. He can cycle at 10 mph; alternatively, he can jog at 6 mph to the bus stop a mile due north of his home, then take the bus which arrives every 5 minutes and takes a direct route to the maths class at 20 mph. If Arav leaves his house at a random time, what is the probability that the bus will get him to class faster than cycling?

12. Jimbo wants to draw a triangle, but he chooses to be a bit chaotic. Every minute, he rolls a standard 6-sided die to help him decide how much of the triangle to draw. If he rolls a 6, he draws all remaining edges of the triangle. If he rolls a 4 or 5, he draws two more edges, unless two edges have already been drawn, in which case he draws just one more edge. If he rolls a 3 or lower, he draws one more edge. What is the probability that Jimbo takes exactly two minutes to draw the triangle?

Set 5

Questions in this set are worth 6 points

13. How many positive integers n have the property that $\frac{1}{n}$ is a recurring decimal with the shortest cycle being 3 digits long?

14. Find the denominator of

$$\frac{1(1+3)(1+3+5)\cdots(1+3+\cdots+23+25)}{20!}$$

when fully simplified.

15. Let $a_1, a_2, a_3, \ldots$ be a sequence of integers with the following properties:

- For every odd positive integer m, $a_m = m$.
- For every positive integer n, the sum of the terms from a_1 up to and including a_n equals the sum of the terms from a_{n+1} up to and including a_{2n}.

Find a_{100}.

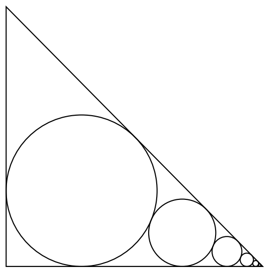
Set 6

Questions in this set are worth 6 points

16. Angelina has four outfits, and each morning, she chooses one of them to wear for the day. She cannot wear the same outfit on two consecutive days. On Sundays, Wednesdays, and Saturdays, she goes on a romantic dinner date with Eren, and so Angelina also cannot wear the same outfit to two consecutive romantic dinners. If Angelina wore her favourite outfit on Saturday, in how many different ways can she choose her outfits for the next week, from Sunday up to and including Saturday?

17. Let a and b be single-digit positive integers, such that $a > b$, and suppose that $a^4 - b^4$ has a total of 6 factors. What is the sum of all possible values of $a^4 - b^4$?

18. Jimbo stumbles upon the fractal shown below. The bottom side of the right triangle has length $\frac{17}{5}$, and the largest circle has radius 1. Assuming that infinitely many circles are drawn following this pattern, what is the sum of the areas of all the circles?



Set 7

Questions in this set are worth 7 points

19. Let $S = \{s_1, s_2, ..., s_n\}$ be the set of all positive factors of 2025. Find the value of

$$\frac{\log_{10}(s_1) + \log_{10}(s_2) + \cdots + \log_{10}(s_n)}{\log_{10}(2025)}.$$

20. Aryan and Bao plan to meet each other on a 3 km long bridge. The bridge has automatic gates on the west and east entrances. The west-side gate opens briefly every 0, 20, and 40 minutes past the hour. The east-side gate opens briefly every 0 and 30 minutes past the hour. Aryan and Bao arrive at opposite ends of the bridge at exactly x minutes past noon, where x is an integer and $0 \leq x < 60$. As soon as the gate opens, each of them begins to walk along the bridge at 2 ms^{-1}. How many values of x allow Aryan and Bao to meet in less than 30 minutes of arriving at the bridge?

21. Julian is ordering pizzas for a very large event. The pizza shop charges him 10 pounds per pizza, and the cost of delivery depends on the size of the order. An order of n pizzas costs $n + 1 - \lfloor\sqrt{n}\rfloor$ pounds to deliver, where $\lfloor\sqrt{n}\rfloor$ represents the largest integer less than or equal to $\sqrt{n}$. Julian must order at least 100 pizzas in total, but he can place multiple orders to be delivered separately. What is the minimum amount that Julian spends on pizzas, in pounds?

Set 8

Questions in this set are worth 8 points

22. Bernie invents a function σ defined by

$$\sigma(s) = \sum_{n=2}^{\infty} \frac{1}{n^s} = \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots$$

Find the value of

$$\sum_{s=2}^{\infty} \sigma(s) = \sigma(2) + \sigma(3) + \sigma(4) + \cdots$$

23. Sam and Owen are stood 200 m apart in a large foggy field. If they ever get within 100 m of each other, they will see each other. Sam and Owen both turn to a random direction and simultaneously begin walking in a straight path. Sam walks at a constant rate of 1 ms^{-1}, and Owen walks at a constant rate of 2 ms^{-1}. What is the probability that they will see each other?

24. Let AB be the diameter of a semicircle, and let X lie on segment AB such that $AX \neq BX$. Choose points Y and Z on the arc of the semicircle such that $AY < AZ$. Let P be the intersection of AZ and XY, and suppose that $AP = 4$, $PX = 2$, $PY = 2$, and $PZ = 3$. Find AY.

Set 9: Estimathon

Questions in this set are worth up to 16 points: where A is the actual answer and E is your estimate for a question, your score is $16 \times \frac{\min(A,E)}{\max(A,E)}$ points, rounded to the nearest integer

25. Let N be the number of positive integer divisors of $2025!$. How many digits does N have?

26. Ivy has a perfectly circular pizza and an atomically precise pizza cutter. Every second, she chooses two random points on the circumference of the pizza and then makes a perfectly straight cut connecting those two points. After she makes 2025 cuts, what is the expected number of pieces of pizza she makes? That is to say, if Ivy were to repeat this procedure sufficiently many times and take the mean of all her results, what number would this mean approach?

27. What is the total number of points scored by all participants in today's OMTC on all questions, except this one?