Oxford Mathematics Team Challenge
Individual Round Question Booklet

Saturday, 8th March 2025

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. **Format.** This round contains 20 multiple choice questions. Cross at most one of the options – A, B, C, D, or E – on the Answer Sheet for each question. Do not cross more than one option.
3. **Time limit.** 60 minutes. You may not write anything into your Answer Booklet, including your Candidate ID, after the allotted time has expired.
4. **No calculators, squared paper or measuring instruments.** Lined paper and blank paper for rough working is allowed. You may use a pen or pencil to preference, however you may want to use a pencil for the Answer Sheet in order to edit your submission. Other mediums for working (e.g., digital devices, whiteboards, trained carrier pigeons) are strictly forbidden.
5. **Scoring rules.** Each candidate starts with 10 points. Each correctly-answered question earns 2 points, whilst each incorrectly-answered question deducts half a point.
6. Don't expect to complete the whole paper in the time! The questions have been arranged roughly in their order of difficulty, with the hardest questions towards the end.
7. You are also encouraged to think deeply, rather than to guess.
8. Good luck, and enjoy! ☺

1. The year 2025 is a square number. How many years is it until the next year which is a square number?

   A. 11; B. 56; C. 91; D. 475; E. 6075.

2. Yvonne has four cards displaying $\boxed{\text{I}}$, $\boxed{\text{I}}$, $\boxed{\text{V}}$, and $\boxed{\text{X}}$. She can use one or more of these cards to form a Roman numeral, e.g. $\boxed{\text{X}}\,\boxed{\text{I}} = 11$. How many distinct valid Roman numerals can she make?

   A. 10; B. 12; C. 14; D. 17; E. 19.

3. Luke is going on a camping trip with a tent in the shape of a square pyramid, with height 4 m and base length 6 m. He needs a tarp to cover the sides of the tent to protect himself from the rain. What is the smallest area of tarp that Luke can buy?

   A. 40 m^2; B. 48 m^2; C. 54 m^2; D. 60 m^2; E. 72 m^2.

4. Linn makes the following claim: ‘A positive integer is prime only if it is one more than a multiple of 4.’ Which of the following positive integers is a counterexample to that claim?

   A. 9; B. 11; C. 13; D. 15; E. 17.

5. For $-90° < x < 90°$, the minimum value of $\tan^2 x - 4\tan x + 5$ is achieved just when the sine of x equals

   A. 0; B. $\frac{2\sqrt{5}}{5}$; C. $\frac{3\sqrt{5}}{5}$; D. $-\frac{1}{2}$; E. 1.

6. Which of the following expressions is negative? *[Note:* $\log x = \log_{10} x$*.]*

   A. $\log\left(2025^{2025}\right)$; B. $\log\left(\log\left(2025^{2025}\right)\right)$;

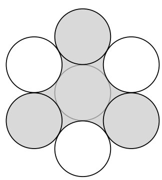
   C. $\log\left(\log\left(\log\left(2025^{2025}\right)\right)\right)$; D. $\log\left(\log\left(\log\left(\log\left(2025^{2025}\right)\right)\right)\right)$;

   E. $\log\left(\log\left(\log\left(\log\left(\log\left(2025^{2025}\right)\right)\right)\right)\right)$.

7. What is the sum of all distinct solutions x to $(x^2 + 6x + 9)^{(2x - x^2)} = 1$?

   A. -5; B. -4; C. 0; D. 4; E. 5.

8. In the diagram shown, each of the seven circles have radius 1 cm. What is the area of the shaded region?

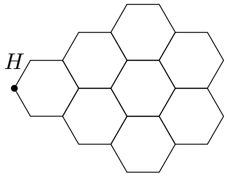
   A. $6\sqrt{3} + \pi$; B. 5π; C. $\frac{27}{2}\sqrt{3}$;

   D. $2\sqrt{3} + 4\pi$; E. $3\sqrt{3} + 3\pi$.

9. The median of the dataset $\{4, 6, 7, 7, 9, x\}$ equals the mean of the dataset $\{4, 6, 7, 7, 9, x, y\}$ where x and y are both positive integers. What is $x+y$?

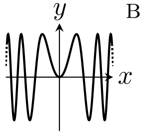
A. 2; B. 6; C. 7; D. 9; E. 16.

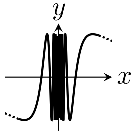
10. On the hexagonal grid with side-lengths 1 km, Hamuul takes a 14 km walk each day which starts and ends at his house at H. On a walk he never travels along the same path twice. How many different daily walks can Hamuul take?

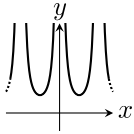


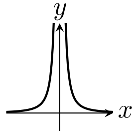
A. 6; B. 7; C. 8; D. 9; E. 10.

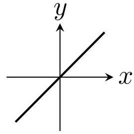
11. Let $p(x) = \sin x$, $q(x) = 1/x$ and $r(x) = x^2$. Cora chooses one of p, q, r, then Derek applies one of p, q, r (possibly the same) to Cora's function. Which of these graphs does Derek's function definitely not look like?

A. 

B. 

C. 

D. 

E. 

12. Aakash is lost on the curve $y = \frac{x}{\sqrt{2}} - \frac{\sqrt{2}}{x}$. On rotating the axes about the origin by a suitable angle, he finds himself at $(1, 1)$. Which of the following points could he have started at?

A. $(0, \sqrt{2})$; B. $(\frac{1}{5}\sqrt{5}, -\frac{3}{5}\sqrt{5})$; C. $(-1, \frac{1}{2}\sqrt{2})$;

D. $(-\frac{1}{3}\sqrt{6}, \frac{2}{3}\sqrt{3})$; E. $\left(\sqrt{2+\sqrt{\frac{8}{3}}}, \sqrt{2-\sqrt{\frac{8}{3}}}\right)$.

13. Let $x_0 = 1$. Which of the following iterative formulae get close to $\sqrt{2}$ as n gets very large?

A. $x_{n+1} = \dfrac{x_n + 6}{3x_n + 1}$; B. $x_{n+1} = \dfrac{2x_n + 1}{x_n + 2}$; C. $x_{n+1} = \dfrac{x_n + 2}{x_n + 5}$;

D. $x_{n+1} = \dfrac{3x_n - 2}{3x_n + 1}$; E. $x_{n+1} = \dfrac{3x_n + 4}{x_n + 2}$.

14. Triangle ABC has lengths $AB = 20$ and $AC = 25$. The midpoint of AB is labelled M, and the midpoint of AC is labelled N. If the circle with diameter BM is tangent to the circle with diameter CN, what is BC?

A. 10; B. 15; C. 22.5; D. 25; E. 30.

15. For a positive integer N, let d be the sum of its digits. We say N is well-fed if $2d < N < 4d$. How many digits can a well-fed number have?

A. 1; B. 2; C. 1 or 2; D. 2 or 3; E. 1, 2 or 3.

16. Peter, Quinn, Rosie and Susan are sat around a round table in some arrangement. They say the following:

Peter: 'I am sat opposite to a liar.'
Quinn: 'I am sat opposite to Peter.'
Rosie: 'I am sat next to at least one liar.'
Susan: 'I am sat opposite to Quinn.'

How many liars can there be?

A. 1; B. 2; C. 1 or 2; D. 1 or 3; E. 2 or 3.

17. A solid sphere of radius 1 cm and a solid sphere of radius R cm can both fit inside a hollow $2 \times 2 \times 2$ cm^3 cube at the same time. What is the largest possible value of R?

A. $\frac{\sqrt{3}-1}{2}$; B. $\frac{\sqrt{3}-1}{1+\sqrt{2}}$; C. $\frac{\sqrt{3}-1}{\sqrt{3}+1}$; D. $\frac{1+\sqrt{3}}{2+\sqrt{3}}$; E. $\frac{1+\sqrt{3}}{1+\sqrt{2}}$.

18. Let f be a polynomial of the form $f(x) = x^4 + ax^3 + bx^2 + cx + d$, where a, b, c, and d are real numbers. Suppose that $f(0) = 0$, $f(1) = 1$, $f(2) = 2$, and $f(3) = 3$. What is the value of $f(4)$?

A. 4; B. 8; C. 16; D. 22; E. 28.

19. Longname's quadrilateral $ABCD$ satisfies $AB = BC = CD = 1$ and $\angle ABC = \angle ADB = 90°$. What is the area of $ABCD$?

A. $\frac{3}{5}$; B. $\frac{3}{8}\sqrt{3}$; C. $\frac{4}{5}$; D. $\frac{1}{2}\sqrt{3}$; E. 1.

20. Your friend is thinking of a function f that takes positive integers, both for inputs and outputs. She tells you that f has the following properties:

- If p is a prime number, then $f(p)$ is also a prime number.
- For all integers $n > 1$, $f(n^2 - 1) = (f(n))^2 + 1$.

Given these properties, there exists a positive integer N such that $f(N)$ must be a 3-digit number. What is N?

A. 24; B. 35; C. 63; D. 80; E. 81.