Oxford Mathematics Team Challenge
Individual Round Solutions

Saturday, 8th March 2025

At the start of the next page is the answer key to the Individual Round, followed by solutions to each question.

Errata

There were no errors in this year's Individual Round.

1	2	3	4	5
C	C	D	B	B

6	7	8	9	10
D	B	A	E	E

11	12	13	14	15
C	D	A	B	B

16	17	18	19	20
B	C	E	A	C

1. $2025 = 45^2$, so the next year which is a square number is 46^2. This happens in $46^2 - 45^2$ years; by the difference of two squares, $46^2 - 45^2 = (46-45)(46+45) = 91$, so the answer is $\boxed{\text{C}}$.

2. Note that 'IIV' is not a valid Roman Numeral! The largest number Yvonne can make is 'XVII' which is 17; there are three numbers she can't make between 1 to 17, namely 3 ('III'), 8 ('VIII') and 13 ('XIII'). So the answer is $\boxed{\text{C}}$.

3. Consider one of the triangular faces of the square pyramid. These have heights of $\sqrt{3^2 + 4^2} = 5$. So the minimum area needed to cover one of these triangular faces is $\frac{1}{2}5 \cdot 6 = 15$. There are 4 such faces, hence the smallest area of tarp that Luke can buy to cover all 4 faces of the pyramid is 60 m^2. The answer is $\boxed{\text{D}}$.

4. We can rephrase Linn's claim to 'The *only way* for a number to be prime is for it to be one more than a multiple of 4.' So we need to find a prime number which isn't one more than a multiple of 4 to disprove Linn's claim. The answer is $\boxed{\text{B}}$.

5. Completing the square, we get $(\tan x - 2)^2 + 1$, so this expression is minimised when $\tan x = 2$. Since $-90° < x < 90°$, we can think of x in a right-angled triangle with the angle $x°$, the opposite side having length 2 and the adjacent length 1. By Pythagoras' Theorem, the hypotenuse has length $\sqrt{5}$, so $\sin x = 2/\sqrt{5} = 2\sqrt{5}/5$. The answer is $\boxed{\text{B}}$.

6. Note $\log(1000) = 3$ and $\log(10000) = 4$, so since log is an increasing function, $3 < \log(2025) < 4$. With this in mind,

   - $\log(2025^{2025}) = 2025 \log(2025) \approx 7000$;
   - $\log(\log(2025^{2025})) \approx \log(7000) \approx 3.5$;
   - $\log(\log(\log(2025^{2025}))) \approx \log(3.5) \approx 0.5$;
   - $\log(\log(\log(\log(2025^{2025})))) \approx \log(0.5) < \log(1)$ and $\log(1) = 0$;
   - Thus $\log(\log(\log(\log(\log(2025^{2025})))))$ is undefined.

   So the answer is $\boxed{\text{D}}$.

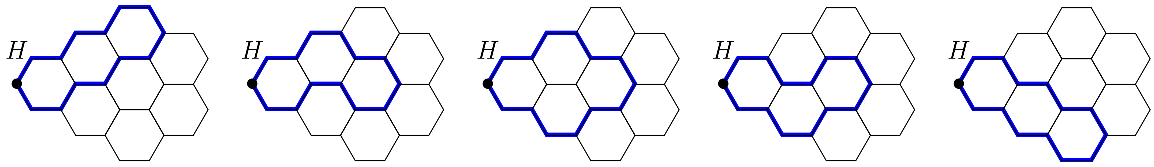
7. The left-hand side expression equals 1 in two cases: either (i) $x^2 + 6x + 9 = 1$; or (ii) $2x - x^2 = 0$ and $x^2 + 6x + 9 \neq 0$. If both the base and the exponent are 0, the expression is undefined!

   In the first case, we get $x^2 + 6x + 8 = 0$, so $(x+4)(x+2) = 0$, so $x = -2$ and $x = -4$ are solutions. In the second case, $x(2-x) = 0$ so $x = 0$ and $x = 2$ are solutions, and neither of them make the base of the expression 0. Thus the sum of distinct solutions is -4, so the answer is $\boxed{\text{B}}$.

8. Let the centres of the outside circles be A, B, C, D, E, F respectively labelled in a clockwise order. Then we know the area of the shaded region is equivalent to the area of $ABCDEF$ + the area of a circle. Now we can see the hexagon is regular with side lengths 2, and hence it has an area of $6\sqrt{3}$. So the total area of the shaded region is $6\sqrt{3} + \pi$ giving us $\boxed{\text{A}}$ as the correct answer.

9. The mean of the dataset $\{4, 6, 7, 7, 9, x, y\}$ is equal to $\frac{33+x+y}{7}$. As we need both x and y to be positive integers, we must have that the median of the data set $\{4, 6, 7, 7, 9, x\}$ is also an integer. Thus we need that $x \geq 7$ (else the median would be 6.5). When $x \geq 7$, the median of $\{4, 6, 7, 7, 9, x\} = 7$ and solving $\frac{33+x+y}{7} = 7$, we obtain $x + y = 16$ so $\boxed{\text{E}}$ is the correct answer.

10. Hamuul needs to go to around one hexagon in the third column of hexagons – any further, and he can't get back in 14 km; any less, and he can't walk far enough without reaching home early or walking along the same path. If Hamuul goes north once he leaves his house, there's five routes he can take:



For each of these paths, he gets an extra daily walk by doing the same in reverse, so he has ten different daily walks. The answer is $\boxed{\text{E}}$.

11. $\sin x$ makes the graph periodic, $1/x$ makes the graph have vertical asymptotes, and x^2 can give "accelerating behaviour" further from the origin, or make the function non-negative. None of these functions can give the properties of the others, so if we find a function with all three properties we know it can't be Derek's function. Indeed, C is periodic (repeating "U" shapes), has vertical asymptotes, and it's non-negative; so the answer is $\boxed{\text{C}}$.

For fun, the graphs are (in order):

$$\sin(x^2), \qquad \sin\left(\frac{1}{x}\right), \qquad \frac{1}{\sin^2(x)}, \qquad \frac{1}{x^2}, \qquad \frac{1}{\left(\frac{1}{x}\right)}$$

(The last graph is just $y = x$, which occurs when Cora and Derek both pick q.)

12. Rotations preserve distance from the origin, so Aakash was essentially on the circle $x^2 + y^2 = 2$. Therefore we want to find the intersections between this circle and the curve. By substitution, we get

$$x^2 + \left(\frac{x}{\sqrt{2}} - \frac{\sqrt{2}}{x}\right)^2 = 2$$

which reduces to the quadratic-in-disguise $3x^4 - 8x^2 + 4 = 0$. This factorises to $(3x^2 - 2)(x^2 - 2) = 0$; the only solution of x that appears in the options is $x = -\frac{1}{3}\sqrt{6}$, and indeed for this value of x, $y = \frac{2}{3}\sqrt{3}$. The answer is $\boxed{\text{D}}$.

13. Let's think about the general formula

$$x_{n+1} = \frac{ax_n + b}{cx_n + d}$$

Assuming this iterative formula converges with $x_0 = 1$, x_n and x_{n+1} get very close as n gets very large, so we can say they both equal some x. In this case,

$$x = \frac{ax + b}{cx + d} \implies cx^2 + (d - a)x - b = 0$$

When does this expression have $\sqrt{2}$ as a root? Just when the equation is of the form $k(x^2 - 2) = 0$, where k is some scaling number. Comparing coefficients, we need $a = d$ and $b = 2c$ (the ratio between the x^2 coefficient and constant term needs to be $1/2$). It's now quick to check that only A satisfies this, so $\boxed{\text{A}}$ is the final answer.

14. Let O_1 be the midpoint of BM and O_2 be the midpoint of CN. Then clearly $4AO_1 = 3AB$ and $4AO_2 = 3AC$ and thus we know $\frac{AO_1}{AO_2} = \frac{AB}{AC}$, and thus O_1O_2 is parallel to BC. But we also know that the point of the tangent of the two circles must lie on the segment O_1O_2. Thus, $O_1O_2 = \frac{1}{4}(AB + AC) = 11.25$. Now as AO_1O_2 is similar to ABC (as O_1O_2 is parallel to BC), we must have $\frac{AO_1}{AB} = \frac{O_1O_2}{BC}$ and hence $BC = \frac{11.25 \times 20}{15} = 15$. So the correct answer is $\boxed{\text{C}}$.

15. We can think of N as $100a+10b+c$, where a, b, c are digits. It follows that $d = a+b+c$.

    Start with the first inequality: if $2d < N$, then $2a + 2b + 2c < 100a + 10b + c$. If N only has one digit, then $a = b = 0$ which implies $2c < c$ – contradiction! So N can't have only one digit.

    For the second inequality: if $N < 4d$, then $100a + 10b + c < 4a + 4b + 4c$. For this inequality to be true we must have $a = 0$, so we can't have three-digit well-fed numbers. Morally we should check that there are in fact any well-fed numbers, and indeed 13 is the smallest well-fed number. The answer is therefore $\boxed{\text{B}}$.

    It turns out there are exactly 13 other well-fed numbers. As an investigation, try to find what their values are.

16. We can split this problem up into how many liars there might be – we just need to see if it's possible for there to be 1 liar, 2 liars, 3 liars.

Suppose there's exactly one liar – could Peter be the liar? If he were, then Rosie must be sat next to him (since she's telling the truth), but now Susan can't be sat opposite to Quinn – a contradiction. If he weren't, then Quinn is either sat next to him and lying (contradiction), or sat opposite him and telling the truth (contradiction). In either case, there's no solution with exactly one liar.

Now suppose there's three liars – could Peter be the liar again? If he were, his statement would certainly check out, but wherever Rosie is sat, she'll be sat next to a liar (which means too many people are telling the truth). If he weren't, then Rosie must be the one telling the truth, meaning she is sat opposite to him. But then Susan is sat opposite to Quinn, so again too many people are telling the truth.

By process of elimination, we know the answer is now $\boxed{\text{B}}$. It turns out that there are two distinct seating arrangements where exactly two of them are lying – as an investigation, try and deduce who the liars are. *[Hint: to get started, Quinn and Susan can't both be telling the truth, and also note that Peter and Quinn can't both be telling the truth – why?]*

17. Intuitively, if we want the sphere of radius R to be as big as possible, we would want to shove the sphere of radius 1 into one corner and the sphere of radius R into the other. As R is as large as possible, we can assume that the spheres touch each other (they touch provided $R < 1$). Now consider a diagonal slice of the cube passing through one of the sphere centres. Clearly this slice also has to contain the other centre and thus must also contain the point of tangency. So we have reduced this problem into 2 dimensions. The distance between the two sphere centres is equal to $1 + R$ and the rectangular slice of the cube has dimensions $2 \times 2\sqrt{2}$. Now consider the right-angled triangle with it's hypotenuse being the segment connecting the two sphere centres and its sides parallel to the sides of the rectangle. Doing some math, we quickly get the sides have lengths $1 - R$ and $\sqrt{2}(1 - R)$ so by Pythagoras we have $3(1 - R)^2 = (1 + R)^2$. Solving this quadratic, we come out with the solution of $R = 2 \pm \sqrt{3}$. Now we know the cube contains both spheres, so $R \leq 2$ and thus $R = 2 - \sqrt{3}$. This is equivalent to C after some rationalizing so $\boxed{\text{C}}$ is the correct answer.

18. Let us construct a new function $g(x) = f(x) - x$. By the conditions given we know that $g(0) = g(1) = g(2) = g(3) = 0$. And thus by the factor theorem we know $g(x) = Q(x)x(x - 1)(x - 2)(x - 3)$ for some polynomial $Q(x)$. Now, the leading coefficient of $f(x)$ is 1 and also $f(x)$ has degree 4, thus we know that $Q(x) = 1$, and so $f(X) = g(x) + x = x(x - 1)(x - 2)(x - 3) + x$. Hence, $f(4) = 28$. So the correct answer is $\boxed{\text{E}}$.

19. Let E be the foot of the perpendicular from C to BD. Now as $BC = CD$, BCD is an isosceles triangle. As $\angle ADB = \angle ABC = 90°$,

$$\begin{aligned}\angle DAB &= 90° - \angle ABD \\ &= \angle DBC \\ &= \angle BDC\end{aligned}$$

But we also know that $AB = BC = CD$ and thus by ASA congruency we can deduce that the triangles DAB, CBE, and CDE are congruent and hence have the same area. Now setting DA to have a length of x, we can deduce that $DB = 2x$ and thus the area of $DAB = x^2$. By Pythagoras we know that

$$\begin{aligned}1 &= AB \\ &= \sqrt{DA^2 + DB^2} \\ &= \sqrt{5x^2}\end{aligned}$$

Hence $x^2 = \frac{1}{5}$. And thus the area of $ABCD = \frac{3}{5}$ so the answer is $\boxed{\text{A}}$.

20. We can find a foothold by considering $n = 2$ in the second property. It gives $f(3) = (f(2))^2 + 1$. Now $f(3)$ and $f(2)$ are prime, but if $f(2)$ is odd, then $(f(2))^2$ is odd so $f(3) = (f(2))^2 + 1$ is even and definitely larger than 2 – contradiction! So $f(2)$ must be even, so $f(2) = 2$ and $f(3) = 5$.

From here, we climb up by choosing good values of n: $f(8) = (f(3))^2 + 1 = 26$, and $f(63) = (f(8))^2 + 1 = 677$. All of the other values aren't fixed from what our friend has told us. The answer is $\boxed{\text{C}}$.