Oxford Mathematics Team Challenge
Maps Round Question Booklet

Saturday, 8th March 2025

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. **Format.** This round contains 48 numerical questions. All answers are positive integers; in your 7×7 grid, write down your answers as **positive integers** in the cells corresponding to the question.
3. **Time limit.** 60 minutes. You may not write anything into your Answer Booklet, including your team name, after the allotted time has expired.
4. **No calculators, squared paper or measuring instruments.** Lined paper and blank paper for rough working is allowed. You may use a pen or pencil to preference, however you may want to use a pencil for the Answer Sheet in order to edit your submission. Other mediums for working (e.g., digital devices, whiteboards, enchanted scrolls) are strictly forbidden.
5. **Scoring rules.** Your team is awarded full points for any correctly-answered questions which are connected to the centre by a series of horizontal and vertical paths of correct answers. Any correctly-answered questions which *aren't* connected are awarded **half** of its full points.

   The points for each question are in the bottom right of the cells in the Answer Sheet, as well as at the end of the questions in the Question Booklet, and are marked in [square brackets].
6. Don't expect to complete the whole paper in the time! The questions further from the centre are worth more marks, however are generally harder.
7. You are also encouraged to think deeply, rather than to guess.
8. To accommodate the online version of the competition, please do not discuss or distribute the paper online until 08:00 BST on Monday 24th March.
9. Good luck, and enjoy! ☺

Column A

A1. Consider a cylinder with an army of 6 ants living on its curved surface. The ants each have a territory, which is the region of points which are less than 1 cm away when distance is measured along the surface. Territories aren't allowed to cross the edges of the cylinder, and – because they don't get along – no two ants' territories can overlap.

Let P, H be its base perimeter and height of the cylinder respectively. The quantity $C = 2(P + H)$ has a minimum value of $a + b\sqrt{3}$ cm, where a, b are integers. What is the value of $a + 10b$? *[Hint: What does C represent geometrically?]* [10]

A2. A circle C centred at $(0, 0)$ has a growing radius which starts at 0 and increases at a constant rate of 10 units per second. The line L starts at $y = -1$ and moves vertically up at a rate or r units per second. The (two) points of intersection between C and L will trace out a curve in the plane. What's the minimum value of r such that this curve is unbounded? [7]

A3. We are trying to wrap a cube of side length 2 by folding a square sheet of paper around it (without cutting it). What's the minimum area of wrapping paper needed to fully wrap the cube? [4]

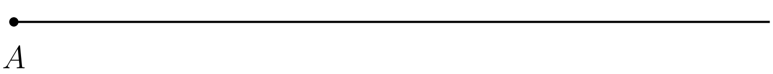
A4. The equation $15x + py = 360$ has p solutions in which x, y are non-negative integers. What is the sum of all possible values of p? [3]

A5. Consider the set S of fractions of the form a/b, for all integer pairs (a, b) which satisfy $0 < a < b < 15$. How many numerically distinct elements does S contain? [4]

A6. A cuboid has side lengths that are all integers in cm, and its surface area is N cm^2, and its volume is N cm^3. What is the maximum possible value of N? [7]

A7. Jason's remote control car travels down a small tube with the aim of getting as far into the tube as possible. The car starts at A with an empty tank. At A there is a reserve of four litres, but the car can only hold one litre of fuel at a time. Jason can give the car the following instructions, any number of times:

- Travel left or right, spending fuel as it moves at a ratio of 1 metre per litre;
- If the car is above a fuel reserve, it can transfer fuel between the car's tank and the reserve;
- The car can set down a fuel reserve at its current location.



The reserves that the car places down can store any amount of fuel. The maximum distance from A that Jason can reach is a/b, simplified as much as possible. What is $a + b$? [10]

Column B

B1. Consider an equilateral triangle ABC of side length 1. Let P be a point in the interior, and consider the region of points within ABC that is closer to P than to any of A, B, C. For different points, P, this region has different areas. The maximum area is M and the minimum area is m. What is the ratio M/m equal to? [7]

B2. A goat is tied at the corner of a 3×4 shed by a rope of length ℓ, where ℓ is a rational number. How long does the rope need to be for the goat to be able to graze in an area 28π? Submit your answer as $a + b$, where $\ell = a/b$ is the simplest fraction for ℓ. [4]

B3. The following 99 people, $P_1, \ldots, P_{99}$, say the following:

$$\begin{array}{rl} P_1: & \pi = 4. \\ P_2: & P_1 \text{ is lying if and only if } P_3 \text{ is.} \\ P_3: & P_2 \text{ is lying if and only if } P_4 \text{ is.} \\ \vdots & \quad \vdots \\ P_n: & P_{n-1} \text{ is lying if and only if } P_{n+1} \text{ is.} \\ \vdots & \quad \vdots \\ P_{98}: & P_{97} \text{ is lying if and only if } P_{99} \text{ is.} \\ P_{99}: & \pi < 4. \end{array}$$

How many of them are lying? [3]

B4. Very fortunately, the triangle ABC has area 24. D is the midpoint of AB and E is the midpoint of AC. Then let M be the midpoint of DE. What is the area of triangle CME? Express your answer as a simplified fraction. [2]

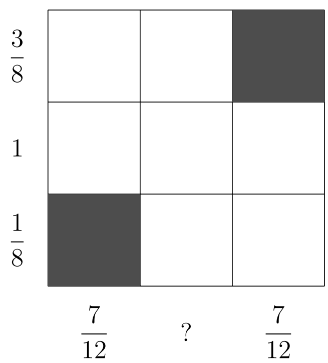
B5. We say a function f is *shrike* if it satisfies the following properties: (i) the domain and range of f is the set $\{1, 2, 3, 4, 5\}$; and (ii) $f(f(x)) = x$ for all values of x in the domain.

How many different shrike functions are there? [3]

B6. With the unit fractions

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{12}, \frac{1}{24},$$

Angus arranges them in the white squares in the grid below, and then writes the sums of the rows and columns as shown in the diagram. What's the reciprocal of the sum of the middle column?



[4]

B7. E_1 starts with the fraction $\frac{1}{2}$. E_2 adds a unit fraction to E_1's fraction to get $\frac{2}{3}$. E_3 adds a unit fraction to E_2's fraction to get $\frac{3}{4}$. This process keeps going until E_n adds $\frac{1}{9900}$. What is n? [7]

Column C

C1. Three circles X, Y, Z having centres A, B, C respectively are externally tangent to each other. Let D on AB, E on BC, F on CA be the intersection points of each pair of circles. Let T_D, T_E, T_F be the circles' tangent lines at points D, E, F respectively.

Suppose the lines T_D, T_E form an angle $90°$, the lines T_E, T_F form an angle $120°$, the lines T_F, T_D form an angle $150°$, and $AB = 5$. Then $BC = \frac{a\sqrt{3}}{b}$ for some positive integers a, b. What is $10a + b$? [4]

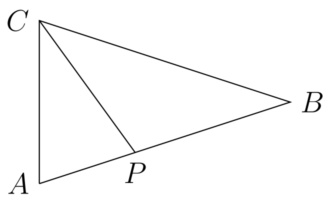
C2. Let $a \neq 0$. How many segments does the curve

$$\frac{1}{x-a} + \frac{x}{x-2a} + \frac{x^2}{x-3a}$$

split the 2D plane into? [3]

C3. There is only one integer n between 1 and 100 such that the sum of the digits of n is half of the sum of the digits of $3n$. What is n? [2]

C4. In the diagram, $AB = BC$, $AC = BP$, $BP = CP$ and $\angle BPC \neq \angle BAC$. What is the angle $\angle ABC$ in degrees?



[1]

C5. When Luci rationalises

$$\frac{2}{\sqrt[3]{3}-1}$$

she gets $\sqrt[3]{a} + \sqrt[3]{b} + c$, where a, b, c are positive integers. What is the value of $a+b+c$? [2]

C6. How many factors of 12! are odd? [3]

C7. The unit fractions $1/a$, $1/b$ and $1/c$ sum to 1, where a, b, c are positive integers. How many different solutions of (a, b, c) are there? *[Note: (a, b, c) is a different solution to, e.g., (b, a, c).]* [4]

Column D

D1. Johnny picks six points in the 2-dimensional plane. He needs to draw at least N straight lines such that any pair of two points is connected by a line. What is the sum of all possible values of N? [3]

D2. How many integer side length triangles are there where two of the sides are length 20 and 25? [2]

D3. What is the sum of the coefficients on the polynomial

$$(1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16})(1+x^{32})$$

including the constant term? [1]

D4. Free space! You don't need to submit anything.

D5. In Agniv's collection, he has 1 lot of 1, 2 lots of 2, ..., and 10 lots of 10. What is the median of Agniv's collection? [1]

D6. What is the value of the smallest positive integer n with the property that there exist digits A, B, and C such that A is non-zero and $n \times AB = ACB$? [2]

D7. Peter has a straight strip of paper 1 cm wide and lays another straight strip 2 cm wide overlapping it. The resulting overlapping region of the two strips of paper has area 4 cm^2. What is the angle between the two strips of paper? [3]

Column E

E1. Rebekah chooses 5 random distinct non-zero digits a_1, a_2, a_3, a_4, a_5 and computes the product $(a_1^{a_1}-1)(a_2^{a_2}-1)(a_3^{a_3}-1)(a_4^{a_4}-1)(a_5^{a_5}-1)$. Let p be the probability that the last digit of the product is 0. What is $1/p$? [4]

E2. A sequence is defined by $u_1 = a$, $u_2 = b$,

$$u_{n+1} = \begin{cases} u_{n-1} + u_n & \text{if } n \text{ is odd} \\ u_{n-1} - u_n & \text{if } n \text{ is even} \end{cases}$$

If the 100th and 101st terms are equal, what is the value of $100 - a/b$? [3]

E3. The triangle PQR satisfies $PQ = PR$. Let S be a point on PQ and T be a point on QR such that $\angle QTS = 90°$. Given that $QT = 20$, that $QS = 25$, and that the area of QST is two ninths the area of PQR, what is the perimeter of PQR? [2]

E4. Find the value of $2 + \sqrt{20 + \sqrt{202 + \sqrt{2025}}}$ rounded to the nearest integer. [1]

E5. A circle is drawn in the plane, and has diameter with endpoints at $P = (2, 8)$ and $Q = (8, 16)$. What is the shortest distance from the origin to a point on the circle? [2]

E6. The graph of $\ln x$ is stretched vertically by a factor b and results in the function $f(x)$. Given that $8^{f(x)} = x^2$ for all x, what is the value of $e^{2/b}$? [3]

E7. A group of 99 people is such that every pair of people have exactly one friend in common. Each person in this group has a *friendship number*, which is just the number of friends that they have. What is the sum of the friendship numbers of people in this group? [4]

Column F

F1. What, from left to right, are the last 4 digits of 11^{2025}? [7]

F2. The digits of 3^{2025} are added up to make a new number. The digits of this new number are added up again to get a third number. We continue this process until we get a single digit number. What is the number? [4]

F3. Rosie, Angie and Ella are standing in a line, with Rosie being at the front and Ella at the back. Each of them are wearing a jersey with a distinct natural number picked from the set $\{1, 8, 3, 100, 2025\}$ on their back, which they cannot see themselves. Each person can see the numbers of all those ahead of them.

First, Ella says that she doesn't know if her number is even or odd. Then Angie says she doesn't know if his number is even or odd. Then Rosie then says she knows whether her number is even or odd.

What are the sum of the numbers Rosie could be wearing? [3]

F4. Let $f(x) = \sin x - \cos^2 x + \sin^3 x - \cos^4 x + \cdots + \sin^{99} x - \cos^{100} x$. Let M be the maximum value of f, and m the minimum value. What is $M - m$? [2]

F5. Let any natural number n which satisfies

$$\sqrt{n+1} - \sqrt{n} < \frac{1}{24}$$

be called a *binky* number. What's the smallest binky number? [3]

F6. A friendly enemy writes the numbers $1, 2, 3, 4, 5$ onto a dark blue wall. Repeatedly, you erase two numbers – call them a and b, with $a > b$ – and write either $2a + b$ or $a + 7b$ onto the wall. This process repeats until there is only one number left. What is the largest number you can create? [4]

F7. Player 67 and Player 456 are playing a game.

They place points on a circle of radius $a > 0$ on each turn. They take turns playing, with Player 67 going first placing a point anywhere on the circle. Thereafter, each player must place a point at distance more than 2π away (as measured on the circle) from all other points. The player who is unable to place a valid point loses. One of these players with number n is guaranteed to win as long as the circle has radius bigger than r. Find $n \times r$. [7]

Column G

G1. Let $d(n)$ be the sum of the digits of n. Given that n equals $2025 \times d(n) \times d(d(n))$, how many prime factors does n have? [10]

G2. The quadratic $2x^2 - 4x + 1$ has the roots α and β, with $\alpha > \beta$. The 2025th digit after the decimal point of α is 2. What is the 2025th digit after the decimal point of β? [7]

G3. When 25! is multiplied by 5^n, it has the highest possible number of zero digits at the end of the number. What is the least value of n? [4]

G4. Given that $z + \sqrt{3} = \sqrt{7 + 4\sqrt{3}}$, what is the value of z^4? [3]

G5. Yoshi starts with a capital of £150. A game consists of a sequence of up to fifty **A**'s and **B**'s, where **A** and **B** are the following actions:

A: Yoshi loses £1.

B: If Yoshi's capital is even, he wins £3. Otherwise, he loses £5.

Yoshi breaks even after a sequence of games if his capital is £150. What is the sum of all game-lengths where Yoshi can break even? [4]

G6. Healy and Dobby are playing 'The Game of 2025', in which they each pick positive numbers, successively adding them to a total. The first player to reach a number at least as big as 2025 loses.

They can pick numbers to add from $\{1, 2, ..., n\}$. If Healy begins, for how many $1 \leq n \leq 7$ does Dobby always have a winning strategy? [7]

G7. Ritter and his evil enemy play a game where a point P starts in a 100×100 cm^2 grid. On each turn, each player moves the point x units right and y units up, where $0 \leq x \leq 1$ and $0 \leq y \leq 1$, with at least one of x, y equal to 1. A player wins once they move the point outside the square. Ritter moves first.

Find the area of the range of starting points in the grid for which Ritter has a winning strategy. [10]